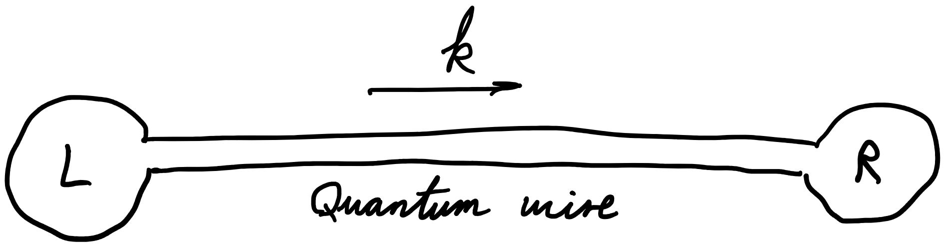
Landauer-Buettiker formalism

Let's consider a long ideal quantum wire



$$v = \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial k}\Big|_{\varepsilon=\varepsilon_F} \quad \text{- Velocity}$$

$$\nu = \frac{\partial n}{\partial \varepsilon}\Big|_{\varepsilon=\varepsilon_F} \quad \text{- DoS, calculated as}$$

$$\frac{dk}{2\pi\hbar} = \nu\, d\varepsilon \rightarrow \nu = \frac{1}{2\pi\hbar v} \quad \text{- DoS}$$

Let's assume the distribution function in the left reservoir is $f_L(k)$, in the right one - $f_R(k)$

Electrons with momenta between k and $k+dk$ (with k directed left); there are $\frac{dk}{2\pi\hbar}$ of them in a unit volume. They contribute current

$$\frac{dk}{2\pi\hbar} v e f(k) = \frac{d\varepsilon}{2\pi\hbar} e f(\varepsilon_k)$$

The total current through the wire

$$I = e\int \frac{d\varepsilon}{2\pi\hbar}\left[f_L(\varepsilon) - f_R(\varepsilon)\right]$$

Applying voltage V to one of the reservoirs

Applying voltage V to one of the reservoirs means the Fermi energy has been increased by eV in the respective reservoir

$$f_L(\varepsilon) = f_R(\varepsilon - eV)$$

Small voltage V

$$I = e\int \frac{d\varepsilon}{2\pi\hbar}\left[f_R(\varepsilon - eV) - f_R(\varepsilon)\right] =$$

$$= -e^2 V \int_{-\infty}^{+\infty} \frac{d\varepsilon}{2\pi\hbar} \frac{\partial f_R}{\partial \varepsilon} = -e^2 V \frac{1}{2\pi\hbar}\left[\underbrace{f_R(\varepsilon = +\infty)}_{=0} - \underbrace{f_R(\varepsilon = -\infty)}_{=1}\right]$$

$$= \frac{e^2 V}{2\pi\hbar}$$

$$\longrightarrow \boxed{G = \frac{e^2}{2\pi\hbar}} \quad \text{(per spin)}$$

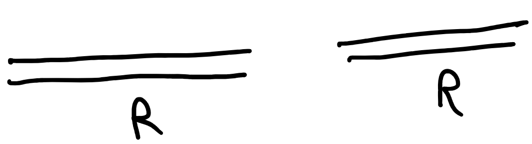
- conductance quantum

Where does the heat go?

Heat $Q = VI = GV^2$

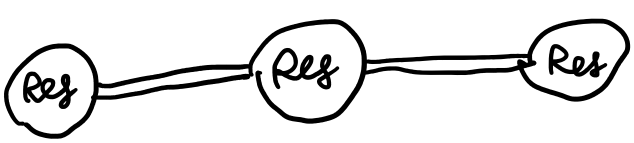
Resistance of an ideal quantum wire $R = G^{-1} = \frac{2\pi\hbar}{e^2}$

of seeming contradiction: the resistance does not depend on the length of the system



(One should expect $R \propto L$)

$R_{total} \neq R_1 + R_2$?



$R_{total} = 2R$

This picture does not work if $L < L_\varphi$

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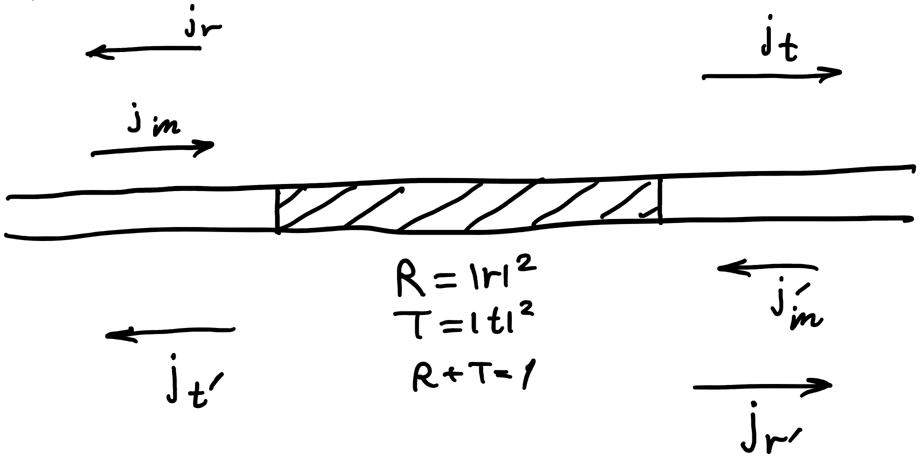
Let's assume the wire is non-ideal

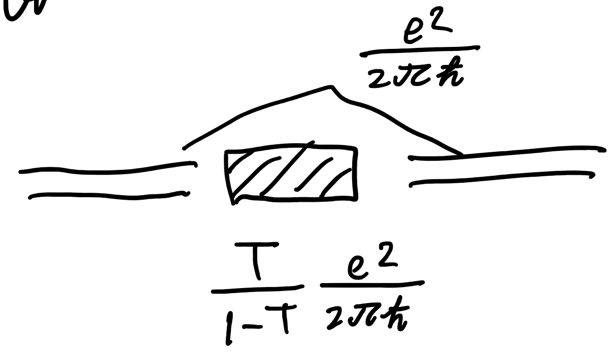


Electrons get through the barrier with probability T

$$I = eT \int \frac{d\varepsilon}{2\pi\hbar} [f_L(\varepsilon) - f_R(\varepsilon)]$$

$$\boxed{G = T \frac{e^2}{2\pi\hbar}}$$ – Landauer formula

A historic remark



$$R = \left(\frac{2\pi\hbar}{e^2}\right) \frac{1-T}{T} + 1 =$$

$$= \frac{2\pi\hbar}{e^2} \frac{1}{T}$$

There may be several channels

*) Spins

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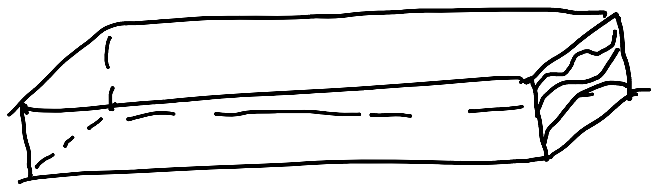
*) Transverse modes



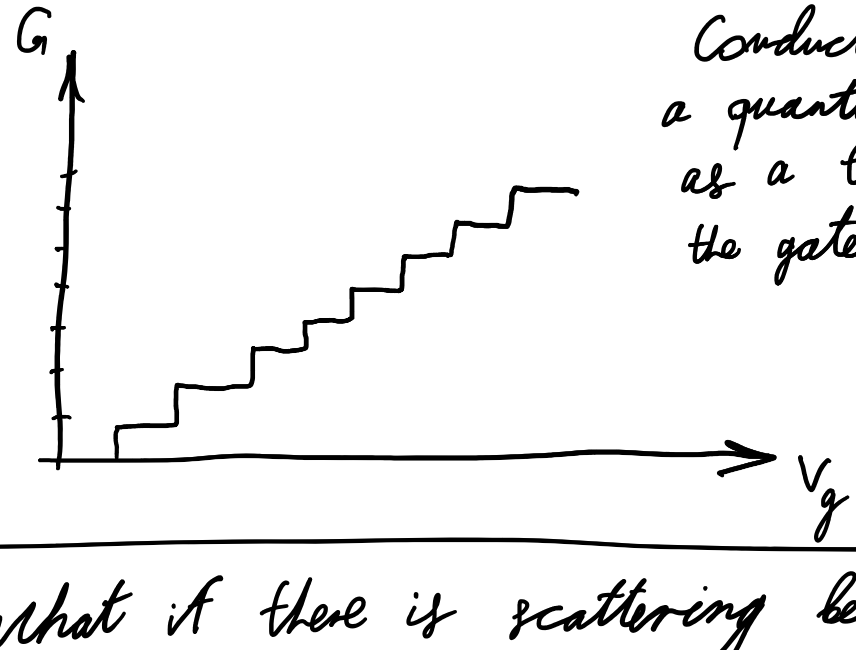
For instance, in a square-cross-section wire

$$\frac{k^2}{2m} + \frac{k_\perp^2}{2m} = E_F$$ - maximal allowed vector k for a given transverse mode $k_\perp$

$$\frac{k^2}{2m} + \frac{n_x^2 + n_y^2}{2\pi^2 m L^2} = E_F$$

Only those modes contribute for which

$$\frac{n_x^2 + n_y^2}{2\pi^2 m L^2} < E_F$$



Conductance of a quantum wire as a function of the gate voltage

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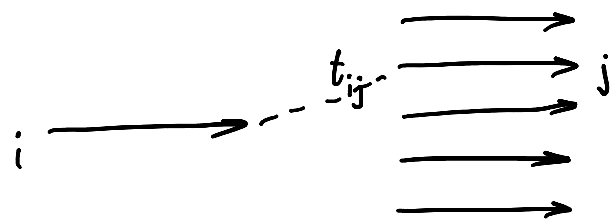
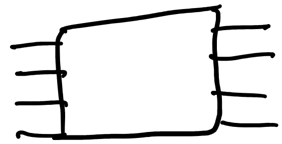
What if there is scattering between channels?

channels !





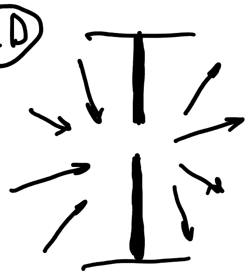
$$G = \frac{e^2}{2\pi\hbar} \sum_i \sum_j |t_{ij}|^2 = \frac{e^2}{2\pi\hbar} \sum_{i,j} t_{ij}\, t^*_{ji}$$

$$G = \frac{2e^2}{2\pi\hbar}\,\mathrm{Tr}\, t t^+$$ – Landauer–Büttiker formula

In principle, this formula may be used to calculate conductance in an arbitrarily complicated disordered system (numerical calculations)

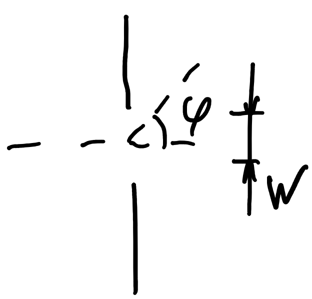
Quantum point contact

2D



Calculating the conductance „classically"

$$I = \delta n \cdot W\, v_F\, e \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\varphi}{2\pi} \cos\varphi =$$



$$= \delta n \cdot W v_F \cdot \frac{1}{\pi} \cdot e$$

$$\delta n = \nu_F\, \delta V e$$

$$G = \frac{1}{\pi} \nu_F v_F W e^2$$

DoS in 2D

$$\frac{2\pi k\, dk}{(2\pi)^2} = \nu\, d\varepsilon \rightarrow \nu_F = \frac{k_F}{2\pi \hbar v_F}$$

$$\frac{2\pi k\,dk}{(2\pi)^2} = \nu\, d\varepsilon \to \nu_F = \frac{k_F}{2\pi\hbar v_F}$$

The conductance of a quantum point contact

$$G = \frac{e^2}{2\pi\hbar}\,\frac{k_F W}{\pi} \quad \text{(per spin)}$$

$$N_\perp = \frac{k_F W}{\pi}$$ – the effective number of transverse modes

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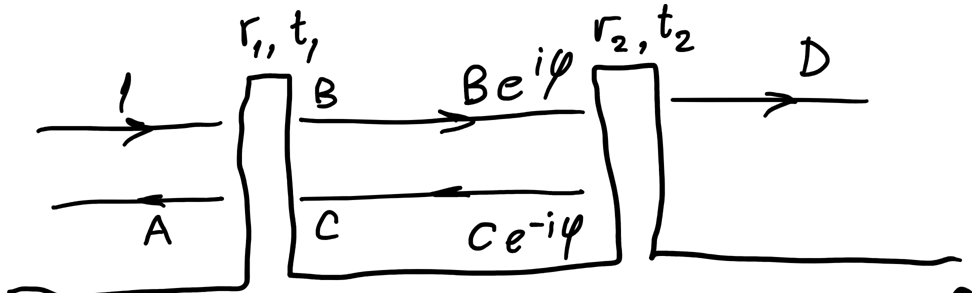
Consider 2 obstacles in a series



$$\begin{cases} A = r_1 + t_1' C \ (1), & B = t_1 + r_1' C \ (3) \\ C e^{-i\varphi} = r_2 B e^{i\varphi} \ (2), & D = t_2 B e^{i\varphi} \ (4) \end{cases}$$

Coeff. for reflection when approaching from the right

$$\left.\begin{matrix}(2)\\(3)\end{matrix}\right\} \to B = t_1 + r_1' r_2 B e^{2i\varphi} \to B = \frac{t_1}{1 - r_1' r_2 e^{2i\varphi}}$$

$$D = \frac{t_1 t_2 e^{i\varphi}}{1 - r_1' r_2 e^{2i\varphi}}$$

$$T = |D|^2 = \frac{T_1 T_2}{1 + R_1 R_2 - 2\sqrt{R_1 R_2}\cos\theta}$$

Resistance

$$R = \frac{2\pi\hbar}{e^2}\,\frac{1 + R_1 R_2 - 2\sqrt{R_1 R_2}\cos\theta}{(1 - R_1)(1 - R_2)}$$

$$R = \frac{2\pi\hbar}{e^2} \frac{1 + R_1 R_2 - 2\sqrt{R_1 R_2}\cos\theta}{(1-R_1)(1-R_2)}$$

If we decided to average wrt the angle θ

$$\tilde{R} = \frac{2\pi\hbar}{e^2} \frac{1 + R_1 R_2}{(1-R_2)(1-R_2)}$$

$$\tilde{G} = \frac{e^2}{2\pi\hbar} \frac{(1-R_2)(1-R_1)}{1 + R_1 R_2} \propto T_1 T_2$$

Consider a chain of barriers

$T_N \approx T_{N-1} \cdot T'$ recurrently

$$T_N \propto e^{-aN}, \quad a = \ln T'$$